Mergeay,[4] for example, demonstrates a considerable degree of uncertainty in the exact number of modes identified in practical tests. Incorrect estimation of the rank of the measured modal matrix will lead either to deletion of genuine structural modes if rank is underestimated or to the identification algorithm inventing its own modes (ghost modes). Either of these errors might lead to serious discrepancies in the adjustment process used by Chen and Fuh. The key to this problem is the use of the weighting matrix $\underset{\sim}{W}$, which Chen and Fuh suggest may be "any symmetric, nonsingular weighting matrix," The weightings should, in practice, reflect the confidence of the analyst in each identified mode.[5]

In conclusion it must be observed that, in using the generalized inverse it is advisable to use proven algorithms. The documentation for the NAg[6] library suggests:

"...only the singular value decomposition gives a reliable indication of rank deficiency....Sound decisions can only be made by somebody who appreciates the underlying physical problem."

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Reply by Authors to J.A. Brandon

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WE wish to thank Dr. Brandon for his interest and comment concerning our Note.[1] Our answers to the posed problems follow.

The first question Dr. Brandon raised concerned the rank of measured modal matrix $\Phi(n \times m)$. In our Note, the full rank assumption was made because we recognized the necessity of engineering judgment in adopting the correct modes before applying model improvement procedures. The rank problem can be viewed from several facets. As far as rank determination is concerned, well-documented numerical algorithms (Gauss elimination, Householder transformation, singular-valued decomposition, etc.) are available for this purpose. The accuracy of rank depends entirely on the algorithm (and specified tolerances) used, provided the modal matrix is not contaminated with errors. Of course, the noise-free environment does not exist in vibration test and modal extraction stages. The measurement uncertainties are expected to be greater than computational errors introduced by the rank determination algorithm. In other words, determination of the rank of a measured modal matrix requires interactive checks and subjective judgment which no "reliable" algorithm can replace.

The basis of our identification method is a well-formulated analytical model and a modal matrix in which we have engineering confidence. Once large model changes result from the improvement procedure, then either the analytical model or Φ, or both, are bad. In order not to digress from our subject, consider the case where the analytical model is a good representation of the structure. An expository account of the relationship between rank and model changes is useful for the understanding of our approach. Let ϕ_i and ϕ_j be two almost, but not exactly, identical vectors of Φ whose differences are large enough to avoid a rank deficiency problem. Normalization of ϕ_i and ϕ_j will lead to $\phi_i^T M_a \phi_i = 1 = \phi_j^T M_a \phi_j$, where M_a is the (unimproved) analytical mass matrix. Obviously, we also have $\phi_i^T M_a \phi_j$ and $\phi_j^T M_a \phi_i$ close to 1. That is, $\Phi^T M_a \Phi$ will make two off-diagonal elements (i,j) and (j,i) approximately equal to 1. However, since the imposed orthogonality condition, $\Phi^T M \Phi = I$ (M, the improved mass matrix), forces all off-diagonal elements to be zero, this constraint would be too rigid and consequently drive the mass changes high. Similar arguments also apply to stiffness changes where the dynamic equation is the imposed constraint. Therefore, we conclude that a "ghost" mode corresponds to large model changes. Physical insight can be gained by using several mode combinations and comparing the associated model changes as made in Ref. 2.

Once the "independent" modes have been selected, they are treated as exact, which is a basic assumption made in the Note. That is, minimum changes on the model should be found to force the improved model exactly satisfying the orthogonality condition and dynamic equation. In mathematical terms, the resultant system is consistent and well defined because the number of degrees of freedom n is much larger than the number of modes of interest m. Following this reasoning, the choice of the weighting matrix does not "reflect the confidence of the analyst in each identified mode" as Dr. Brandon believes and, thus, has nothing to do with rank deficiency. The method in the Note is quite different from the ordinary least-square approach where the confidence level in each mode is used to adjust the error distribution and may have significant effects on the improved model. The weighting matrix given in the Note, however, is (in a sense) used to introduce the confidence level in each element that is subjected to change during the minimum-norm identification procedure. Dr. Brandon may have overlooked the difference between these two approaches, i.e., minimum-norm vs least-square.

In the last paragraph of his Comment, Dr. Brandon seemed to suggest that the singular valued decomposition (SVD) is the only reliable algorithm in computing the Moore-Penrose inverse A^+ of an $n \times m$ matrix of rank r. SVD is well known for its numerical stability. Unfortunately, since it invokes the eigen solutions of an $n \times n$ matrix, AA^T, and an $m \times m$ matrix, $A^T A$, SVD is very inefficient for analytical model improvement where n is of hundreds or thousands. A much more efficient and equally powerful algorithm is the Householder transformation, used to decompose A into the form QU, where Q is an $n \times r$ orthogonal matrix and U an upper triangular.[3] Then, the Moore-Penrose inverse A^+ reads $U^T(UU^T)^{-1}Q^T$. This algorithm gains its efficiency and reliability from the following: 1) complete information of A^+ is obtained from decomposition of A (no products like AA^T or $A^T A$ in SVD are required); 2) the only regular inverse performed is UU^T, a matrix of order r; and 3) the Householder transformation is numerically stable and the factored matrices can be efficiently stored.

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Comment on "Laminar Stagnation-Point Heat Transfer for a Two-Temperature Argon Plasma"

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IN Ref. 1, the authors attempt to simplify the heat transfer problem by assuming that "the freestream static temperature near the stagnation point is constant along the x direction and is equal to that at the stagnation point." While they considered the Joule heating term to be constant, as shown in Eq. (16), they neglected to balance the global energy in the freestream. If the above assumption is used in the global energy equation, the current density should be zero in the freestream. And if the Joule heating in the boundary layer is constant, as they have assumed, then it should be zero. On the other hand, if the externally applied electric field is a nonzero constant, then there should be additional terms in the energy equations from (18) through (20) pertaining to the freestream temperature variation along the x direction.

It is apparent that a stream function ψ is used so that the continuity equation is satisfied by $\rho u r_w = \partial\psi/\partial y$ and $\rho v r_w = -\partial\psi/\partial x$. However, the stream function is not defined, nor is the velocity component in the y direction defined in terms of the "dimensionless normal velocity." The sign for the term $\rho_0 U_0 \mathrm{d}U_0/\mathrm{d}x$ in Eq. (9) is incorrect.[2] This term is then dropped completely from Eq. (17), which does not, therefore, represent the flow configuration considered in the paper.

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Reply by Authors to H. Chuang

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IN response the Comment, we would like to point out some errors in the paper.[2] First, the freestream velocity, U_0, in Fig. 1 is to be replaced by U_∞, the approaching flow velocity. For an axisymmetric spherical shaped body, the relation between the two in the stagnation region is $U_0 = 3U_\infty x/(2R_w)$. Second, as pointed out in by Chuang, the sign in the first term on the right-hand side of Eq. (9) is positive. Third, in Eq. (4), x_i is replaced by x_e.

The term $\rho_0 U_0 \mathrm{d}U_0/\mathrm{d}x$ in Eq. (9) can easily be transformed to a pressure gradient parameter $\beta = (2s/U_0)(\mathrm{d}U_0/\mathrm{d}x)$, which, as is well-known, has a value of ½ for the present case.[3] This value is used in the third term of Eq. (17). In addition, because of the above freestream velocity distribution in the stagnation region, $U_0 = 0$ as $x \to 0$. Furthermore, at the stagnation point, $\partial h/\partial x = 0$ due to symmetry. Thus, in the freestream the first term in Eqs. (10) and (12) is zero, and it is left to the other y-dependent terms to balance the Joule heating term in the freestream. The obvious conclusion is that the $\partial/\partial y$ terms in these two equations are not zero, unless the Joule heating term in the freestream is balanced by a term due to a dissipative mechanism such as radiation, which has not been included in the present study. However, the problem is not serious, since in a differential equation of second order only two boundary conditions are to be prescribed—the temperatures at $\eta = 0$ and η_{max}—and nonzero values of the temperature gradients in the freestream need not be taken into account. Actual numerical calculation shows that this gradient at the freestream ($\eta = \eta_{max}$) for the current density range studied in the paper[1] is indeed very small.

As Chuang correctly surmises, the transformation of the continuity, momentum, and energy equations from the body-based coordinate system to those in the (s,η) coordinate system requires introduction of stream function and "dimensionless normal velocity." These definitions were thought to be quite standard, and were left out to save space. However, Refs. 2 and 3 will give details about these definitions.

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Comment on "Dynamic Condensation"

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IN Ref. 1, Paz has pointed out some of the shortcomings of static condensation methods such as those of Guyan[2] and Irons[3] when used to reduce the number of degrees of freedom considered in vibration analysis and dynamic response analysis of systems having large numbers of degrees of freedom. He has presented a technique of dynamic condensation that greatly improves the accuracy of both eigenvalues and eigenvectors for such methods. Moreover, in principle, the dynamic condensation method, iteratively applied, can lead to exact solutions for the eigenvalues and eigenvectors of the complete unreduced matrix equation subject only to the usual limitations of computational accuracy.

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